

Beijing City Senior High School Entrance Examination 2008

Mathematics Examination Paper

Notice to Candidates	<ol style="list-style-type: none">1. This examination paper is divided into Paper I and Paper II. Paper I has 2 pages in total, and Paper II has 8 pages in total. There are 9 long questions and 25 short questions in this paper.2. The total score for this examination paper is 120 points, and the time limit for the examination is 120 minutes.3. Write down the name of your district (county), the name of your school, your name, Registration No. and Admission Card No. accurately below the end line of the examination paper (including Paper I and Paper II).4. Return this paper together with the answer sheet upon completion of the examination.
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Paper I (Machine-readable Paper, 32 points in total)

Notice to Candidates	<ol style="list-style-type: none">1. Paper I starts at page 1 and ends at page 2. There are 2 pages in total. There is 1 long question and 8 short questions.2. Candidate, please insert your selected options on the answer sheet according to the requirements. Any answers that are written on the examination paper shall be considered invalid.
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I. Multiple Choice (8 short questions in total, 4 points for each short question, 32 points in total)

Each of the following questions has four options, and only one option is the correct answer. Use a pencil to mark the blank boxes with the answers corresponding to the questions on the “Machine-readable Answer Sheet.”

1. The absolute value of -6 is equal to:
A. 6 B. $\frac{1}{6}$ C. $-\frac{1}{6}$ D. -6
2. As of 19 May 2008, 21,600 Chinese and foreign reporters became registered reporters of the Beijing Olympics, amounting to the greatest number of reporters in the all previous Olympic Games. The scientific number 21,600 is expressed as:
A. 0.216×10^5 B. 21.6×10^3 C. 2.16×10^3 D. 2.16×10^4
3. If the radii of two circles are 1cm and 5cm respectively, and the centre distance is 6cm, then the positional relationship between these two circles is:
A. internally tangential B. intersecting C. externally tangential D. separated
4. Collective efforts are being made to bring aid to earthquake victims. A small group of 7 students enthusiastically donated their pocket money to the area hit by the disaster. Their donations were made in the amounts of (unit: dollar): 50, 20, 50, 30, 50, 25 and 135 respectively. The mode and median of this group of numbers are:
A. 50, 20 B. 50, 30 C. 50, 50 D. 135, 50

5. If the sum of the interior angles of a polygon is equal to 720° , then the number of sides of the polygon is:
 A. 5 B. 6 C. 7 D. 8
6. The 5 cards shown below are of the same shape, size and quality. The front of the cards are printed with 4 different patterns, namely the emblem, lucky dolls (Fuwa), torch and medals of the Beijing Olympics. Their reverse sides of the cards are identical to the front of the cards. The 5 cards are shuffled, and then placed on the table with the front side facing downwards. Remove one card at a time until the lucky dolls (Fuwa) is:

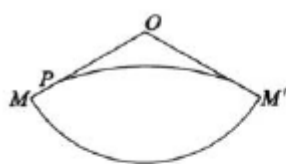


- A. $\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{1}{2}$ D. $\frac{3}{5}$

7. If $|x+2| + \sqrt{y-3} = 0$, then the value of xy is

- A. -8 B. -6 C. 5 D. 6

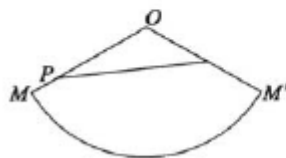
8. We know that O is the vertex of a cone, M is a point on the lower circle of the cone, and point P is on OM . A snail starts walking from point P around the lateral side of the cone and returns to point P , with the track of the shortest route shown on the right figure. If the lateral side of the cone is cut along OM and the cone is spread out, the spread diagram of the lateral side is:



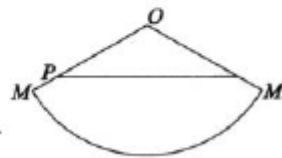
A



B



C



D

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Mathematics Examination Paper

Paper II (Non-machine-readable Paper, 88 points in total)

Notice to Candidates	1. Paper II starts at page 1 and ends on page 8, with 8 pages in total. There are 8 long questions and 17 short questions. 2. Apart from diagrams that can be drawn in pencil, the candidate must use black or blue ballpoint or fountain pen to answer the questions.
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Question No.	II	III	IV	V	VI	VII	VIII	IX	Total Points
Points									
Examiner									
2 nd Examiner									

II. Fill in the blanks (4 short questions in total, 4 points for each short question, 16 points in total)

9. In the function $y = \frac{1}{2x-1}$, the value range of the independent variable x is

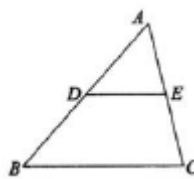
_____.

Points

10. Divide the following into factors: $a^3 - ab^2 =$ _____.

Points

11. In the figure, $\triangle ABC$, D and E are the middle points of AB and AC respectively. If $DE = 2\text{cm}$, then BC



$=$ _____ cm.

Points

12. Here is a group of formulae in the correct order: $-\frac{b^2}{a}, \frac{b^5}{a^2}, -\frac{b^8}{a^3}, \frac{b^{11}}{a^4}, \dots (ab \neq 0)$,

where the 7th formula is _____, and the n^{th} formula is _____ (n is a positive numeral).

Points

III. Problem solving (5 short questions in total, 25 points in total)

13. (The total score for this short question is 5 points.)

Points

Calculate: $\sqrt{8} - 2 \sin 45^\circ + (2 - \pi)^0 - \left(\frac{1}{3}\right)^{-1}$.

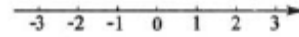
Solution:

14. (The total score for this short question is 5 points.)

Points

Solve the inequality $5x - 12 \leq 2(4x - 3)$, and insert the solution along the number axis.

Solution:

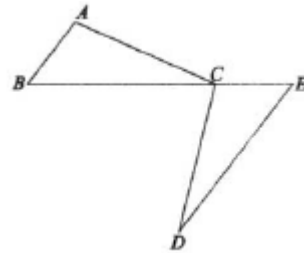


15. (The total score for this short question is 5 points.)

Points

As known in the figure, C is a point on BE . Points A, D are by the two sides of BE . $AB \parallel ED, AB = CE, BC = ED$. Prove $AC = CD$.

Proof:

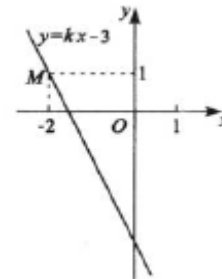


16. (The total score for this short question is 5 points.)

Points

In the figure, we know that the straight line $y = kx - 3$ goes through point M . Find the coordinates of the intersecting points between the straight line and axes x and y .

Solution:



17. (The total score for this short question is 5 points.)

Points

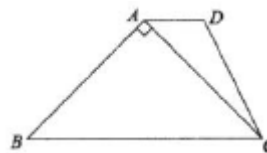
We know that $x - 3y = 0$. Find the value of $\frac{2x + y}{x^2 - 2xy + y^2} \cdot (x - y)$.

Solution:

IV. Problem-solving (2 short questions in total, 10 points in total)

18. (The total score for this short question is 5 points.) Points

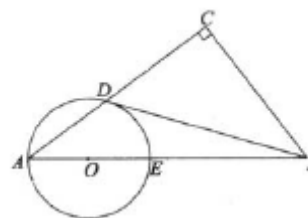
As shown in the figure, in the trapezium $ABCD$, $AD \parallel BC$, $AB \perp AC$, $\angle B = 45^\circ$, $AD = \sqrt{2}$, $BC = 4\sqrt{2}$. Find the length of DC .



Solution:

19. (The total score for this short question is 5 points.)

As known in the figure, in $\text{Rt } \triangle ABC$, $\angle C = 90^\circ$, and point O is on AB . O is the centre of the circle, and OA is its radius. The circle intersects AC , AB at points D , E respectively, and $\angle CBD = \angle A$.



- (1) Judge the positional relationship between the straight line BD and $\odot O$, and then prove your conclusion;
- (2) If $AD : AO = 8 : 5$, $BC = 2$. Find the length of BD .

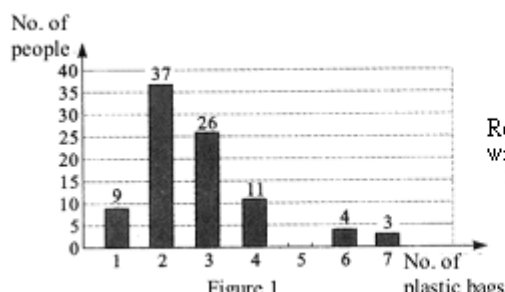
Solution: (1) Points

(2) points

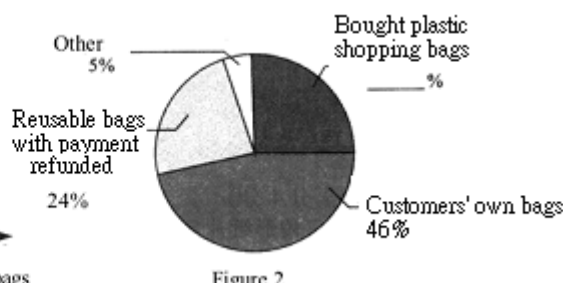
V. Problem-solving (The total score for this problem is 6 points.)

20. For the sake of environmental protection, as from 1 June 2008 retail shops throughout the nation began the implementation of the “Plastic Bag Compensated Use System” (hereinafter referred to as “Reduced Plastic”). One day before the middle of June, a class of students conducted a survey in front of a supermarket and, at random, investigated customers’ uses of plastic bags inside the supermarket before and after the implementation of “Reduced Plastic.” Below is part of the statistical table drawn according to 100 valid questionnaires filled out by 100 customers:

Statistical chart of the number of people using different numbers of plastic shopping bags on average during one shopping trip before the implementation of “Reduced Plastic”



Statistical distribution chart of people using different kinds of shopping bags after the implementation of “Reduced Plastic”



Statistical table of handling ways of the used plastic shopping bags after implementation of “Reduced Plastic”

Handling way	Directly discarded	Directly used as garbage bag	Reused as shopping bag	Others
% of people choosing this item	5%	35%	49%	11%

Please answer the following questions according to the above information:

- In addition to Figure 1, before the implementation of “Limit Plastic Order”, if approximately 2,000 trips are made to the supermarket to shop every day, according to the average number of plastic shopping bags used by these 100 customers during one shopping trip, estimate the number of plastic shopping bags that the supermarket must provide to customers every day;
- In addition to Figure 2, and according to the explanation of the statistical chart and table, how should customers choose their shopping bags? What should be done with the used plastic bags in order to promote environmental protection?

Solution: (1) Points

(2) Points

VI. Problems (2 short questions in total, 9 points in total)

21. (The total score for this short question is 5 points.) **List out equations or a system of equations to solve this problem:**

Beijing-Tianjin cross-city railway shall start running as from 1 August 2008. It is expected that a single direct journey on the high-speed train running between Beijing and Tianjin takes half an hour. On a trial run, the time it took the test train to run from Beijing to Tianjin was 6 minutes more than the expected time, and the return journey from Tianjin to Beijing takes the same amount of time as the expected time. Presume that in this trial run, the return journey from Tianjin to Beijing is faster than the journey to Beijing by 40 kilometers per hour on average. What, therefore, is the average speed in kilometers per hour for the journey from Beijing to Tianjin on this trial run?

Solution:

Points

22. (The total score for this short question is 4 points.)

We know that one side of a piece of isosceles triangular paper ABC is 8. D is a point on side AB . A line is drawn from point D to make $DG \parallel BC$, and touch AC at point G . $DE \perp BC$ at point E . A line is drawn from point G to make $GF \perp BC$ at point F . Fold the piece of isosceles triangular paper ABC along DG , DE , DG , in the way as shown in Figure 1. Points A, B, C lie on A', B', C' respectively. If points A', B', C' are inside the rectangle $DEFG$ or on its sides and do not overlap, then we can call $\triangle A'B'C'$ (i.e. the shaded part in the figure) an “overlapping triangle.”

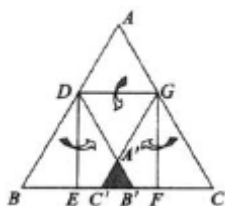


Figure 1

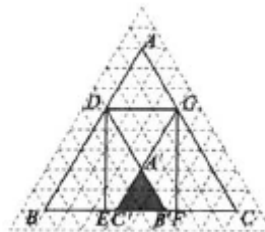
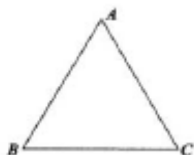


Figure 2

- (1) If the piece of triangular paper ABC is placed inside the grid diagram of an isosceles triangle (in the grid diagram, the length of a side of each small equilateral triangle is 1). Points A, B, C, D exactly fall on the grid points inside the grid diagram, as shown in Figure 2. Please directly write down the area of the overlapping triangle $A'B'C'$;
- (2) Experimental investigation: Suppose that the length of AD is m . If the overlapping triangle $A'B'C'$ exists, use an algebraic expression in terms of m to express the area of the overlapping triangle $A'B'C'$, and write down the value range of m (directly write down the result, and the prepared diagrams are for the use of experimental investigation).



Prepared diagram



Prepared diagram

Solution: (1) The area of the overlapping triangle $A'B'C'$ is

Points

(2) The area of the overlapping triangle $A'B'C'$ indicated by an algebraic expression in terms of m is _____;

the value range of m is _____. Points

VII. Problem-solving (the total score for this problem is 7 points)

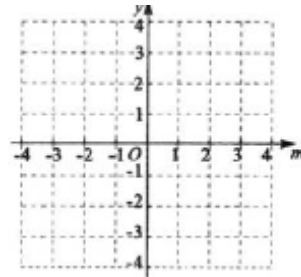
23. We know that there is a quadratic equation in terms x : $mx^2 - (3m + 2)x + 2m + 2 = 0$ ($m > 0$).

- (1) Prove: The equation has two unequal real number roots;
- (2) Let the two real number roots of the equation be x_1, x_2 (where $x_1 < x_2$). If y is a function in terms of m , and $y = x_2 - 2x_1$, find the analytic expression of the function;
- (3) Under the condition of (2), together with the function on the graph, answer this question: In what condition the value of the independent variable m should satisfy to make $y \leq 2m$ exist?

(1) **Proof:** Points

(2) **Solution:** Points

(3) **Solution:** Points

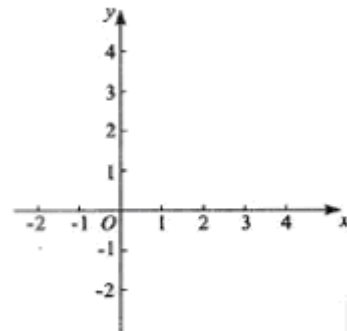


VIII. Problem-solving (The total score for this problem is 7 points.)

24. In a plane rectangular coordinate system xOy , the parabola $y = x^2 + bx + c$ intersects with x axis at points A, B (point A is by the left hand side of point B), and intersects with y axis at point C . The coordinates of point B are $(3, 0)$. Move the straight line $y = kx$ upwards along y axis to a length of 3 units and let it pass through the two points B, C .

- (1) Find the analytic expressions of the straight line BC and the parabola;
- (2) Suppose the vertex of the parabola is D , point P lies on the symmetric axis of the parabola, and $\angle APD = \angle ACB$. Find the coordinates of point P ;
- (3) Connect CD . Find the sum of the two angles, $\angle OCA$ and $\angle OCD$.

Solution: (1) Points



(2) Points

(3) Points

IX. Problem-solving (The total score for this problem is 8 points.)

25. Read the following material:

Question: As shown in Figure 1, in rhombus $ABCD$ and rhombus $BEFG$, points A, B, E are on the same straight line, and P is the middle point of the line segment DF . Connect PG, PC . If $\angle ABC = \angle BEF = 60^\circ$, find the relationship between position PG and PC , and the value of $\frac{PG}{PC}$.

Xiao Cong has an idea in his mind that extending GP to intersect DC at point H will form a congruent triangle. After inference, the problem is solved.

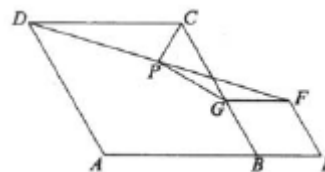


Figure 1

(1) Write down the relationship between position

PG and PC , and the value of $\frac{PG}{PC}$;

(2) Rotate point B of rhombus $BEFG$ in Figure 1 in clockwise direction, and let diagonal BF of rhombus $BEFG$ lie on the same straight line of side AB of rhombus $ABCD$, with all other conditions in the original question remaining unchanged (as shown in Figure 2). Are there any changes to the two conclusions that you made in (1)? Write down your estimate and show how you arrived at this conclusion.

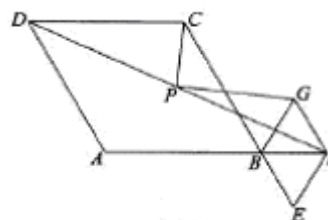


Figure 2

(3) In Figure 1, if $\angle ABD = \angle BEF = 2\alpha$ ($0^\circ < \alpha < 90^\circ$), point B of rhombus $BEFG$ is rotated in clockwise direction for a random angle, with all other conditions in the original question remain unchanged. Please directly

write down the value of $\frac{PG}{PC}$ (shown in an expression in terms of α).

Solution: (1) The positional relationship between PG and PC is

_____;

$\frac{PG}{PC} =$ _____.

(2)

(3) $\frac{PG}{PC} =$ _____.